Engineering Notes

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Weight Criterion on Flow Control in Level Flight

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Introduction

FLOW control may potentially offer a tremendous economical benefit, particularly in aviation. Extensive effort has been made to understand the physics of flow control, development of various actuators, control methodology, wind-tunnel tests, and computational fluid dynamics simulations [1,2]. However, the net benefit of a flow control system that adds an extra weight to an aircraft in flight has not been quantitatively evaluated, although people are always aware of the weight penalty. To address this basic problem, this note gives a weight criterion for flow control in level flight by assessing a reduction of the power required for flight by a flow control system. The weight criterion is applied, as an illustrating example, to drag reduction by normal injection, and the benefit margin is given as a function of a number of the parameters such as the mean aircraft weight, relative control system weight, pressure difference, injection area ratio, and control power density ratio.

Weight Criterion

The power required for level flight is [3]

$$P_R = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S_{\text{wing}} C_L^3}} \tag{1}$$

where W is the aircraft weight, S_{wing} is the wing area, C_D is the drag coefficient, and C_L is the lift coefficient. Flow control on an aircraft usually changes both lift and drag, at a cost of adding a weight ΔW to the aircraft. Differentiation of Eq. (1) and use of a scaling law $S_{\text{wing}} = aW^{2/3}$ yield a reduction of the power required for level flight by flow control:

$$\Delta P_R = \frac{bW^{7/6} C_D}{C_L^{3/2}} \left(\frac{7}{6}\frac{\Delta W}{W} + \frac{\Delta C_D}{C_D} - \frac{3}{2}\frac{\Delta C_L}{C_L} \right) \tag{2}$$

where $b = \sqrt{2/a\rho_\infty}$ and ΔW is the total weight of a control system including actuators, power units, and associated structures. Equation (2) clearly indicates the effects of the weight increase, drag reduction, and lift enhancement on the power required for flight.

To achieve the net benefit, the actuating power $P_{\text{FC},r}$ required for flow control should be smaller than the power reduction by flow control (i.e., $\Delta P_R + P_{\text{FC},r} < 0$). From Eqs. (1) and (2), we obtain a weight criterion for flow control

$$-\frac{6}{7}\frac{\Delta C_D}{C_D} + \frac{9}{7}\frac{\Delta C_L}{C_L} - \frac{6}{7}\frac{P_{\text{FC},r}}{P_R} - \frac{\Delta W}{W} > 0 \tag{3}$$

The relative control system weight $\Delta W/W$ is constrained by the relative drag reduction, lift enhancement, and actuating power required for flow control. Further, introducing the actuating power density $\eta_{\text{FC},r} = P_{\text{FC},r}/\Delta W$ required for flow control, we have

$$\left[1 + \frac{6}{7}\left(\frac{\eta_{\text{FC},r}}{b}\frac{C_L^{3/2}}{C_D} \right) W^{-1/6} \right]^{-1} \left(-\frac{6}{7}\frac{\Delta C_D}{C_D} + \frac{9}{7}\frac{\Delta C_L}{C_L} \right) - \frac{\Delta W}{W} > 0 \tag{4}$$

The form of Eq. (3) or Eq. (4) is generic and applicable to any aircraft. The control system weight limit can be estimated as long as the drag reduction, lift enhancement, control power density, and lift and drag coefficients are given. In a parametric study, ΔC_D and ΔC_L are functions of $\Delta W/W$, W, flow parameters, and control parameters for a specific flow control system employed in a particular aircraft.

The drag coefficient in Eq. (4) can be further decomposed into the zero-lift drag and induced drag, that is, $C_D = C_{D0} + KC_L^2$, where C_{D0} is the zero-lift drag coefficient and $K = (\pi e AR)^{-1}$ is inversely proportional to the Oswald efficiency e and wing aspect ratio AR. Therefore, the term in Eq. (4) is rewritten as

$$-\frac{6}{7}\frac{\Delta C_D}{C_D} + \frac{9}{7}\frac{\Delta C_L}{C_L} = -\frac{6}{7}\frac{\Delta C_{D0}}{C_D} + \frac{9}{7}\frac{\Delta C_L}{C_L}\left(1 - \frac{12}{9}\frac{KC_L^2}{C_D} \right) \tag{5}$$

The first term in the right-hand side of Eq. (5) is the zero-lift drag reduction, and the second term is the contribution of the lift enhancement minus the associated induced drag penalty. To keep the second term positive, another criterion is $KC_L^2/C_D < 9/12$. For most aircraft with a reasonably large wing aspect ratio, this criterion is satisfied in cruise flight. In an ideal case of passive flow control in which $\eta_{\text{FC},r} = 0$ and $\Delta W/W$ can be neglected, the weight criterion is reduced to

$$-\frac{6}{7}\frac{\Delta C_{D0}}{C_D} + \frac{9}{7}\frac{\Delta C_L}{C_L}\left(1 - \frac{12}{9}\frac{KC_L^2}{C_D} \right) > 0 \tag{6}$$

Here, the weight criterion is given based on a reduction of the power required for level flight. The objective function based on the power required is reasonable for flow control in level flight, because it directly leads to reduced fuel consumption and longer range and endurance. The proposed approach can be applied to other cases for which the objective functions of flow control are different. For separation control to reduce the stall speed $V_{\text{stall}} = bW^{2/3}C_{L\max}^{-1/2}$ at high angles of attack, for example, a weight criterion $(3/4)(\Delta C_{L\max}/C_{L\max}) - \Delta W/W > 0$ can be similarly given. In some cases, multiple weight criteria could be imposed.

Drag Reduction by Normal Injection

To illustrate the use of the weight criterion, we consider a classical technique: viscous drag reduction by normal mass injection. Because of the local modification of the velocity profile of a boundary layer by

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normal air (gas) injection through a porous/perforated wall, the skin friction is decreased. This technique is feasible to boundary layers with a zero pressure gradient on a flat plate and cylindrical body (e.g., the main portion of a fuselage). Over a considerable range of Mach numbers ($0 < M < 0.6$), the skin friction coefficient for normal injection can be expressed by $C_f/C_{f,0} = f(\xi)$ [4], where $\xi = 2\rho_w V_w/(C_{f,0}\rho_e V_e)$ is the normalized injection velocity, V_w and V_e are the normal injection velocity at the wall and external velocity around a body, respectively, ρ_w and ρ_e are the densities at the wall and external flow, respectively, and $C_{f,0}$ is the skin friction coefficient with no injection. It has been shown that experimental and computational data for drag reduction at different velocities are collapsed into a single function that can be reasonably given in an empirical expression $f(\xi) = \exp(-\xi/2)$ for $0 \le \xi \le 3$.

The net skin friction drag reduction by normal injection is

$$\begin{aligned}\Delta D_f &= \frac{1}{2}\rho_\infty V_\infty^2 \int_{S_w} (C_f - C_{f,0})\,\mathrm{d}S \\ &= \frac{1}{2}\rho_\infty V_\infty^2 \int_{S_w} C_{f,0}[f(\xi) - 1]\,\mathrm{d}S\end{aligned} \tag{7}$$

where S_w is the area of the porous/perforated wall. When the normal injection technique is applied to a cylindrical fuselage in which the pressure gradient is small and the skin friction coefficient gradually varies along the surface, replacing $C_{f,0}$ in Eq. (7) by an average value $\bar{C}_{f,0}$, we have an estimate for the coefficient of the net skin friction drag reduction $\Delta C_{D_f} = (S_w/S_{\mathrm{wing}})\bar{C}_{f,0}[f(\bar{\xi}) - 1]$, where $\bar{\xi} = 2\rho_w V_w/(\bar{C}_{f,0}\rho_e V_e)$ is a mean value of the normalized injection velocity.

Although the injection technique can reduce the skin friction drag, additional penalty associated with taking injection air to board should be considered. Two cases are considered here. In the first case, injection air is provided by inlets, which produces additional ram drag. In the second case, air suction is applied to the rear part of a fuselage to provide injection air, which may also reduce the pressure drag. However, vacuum pumps and associated equipment for air suction consume more power and add more weight. Note that the injection technique is discussed here mainly for demonstrating the application of the proposed weight criterion, although it may not be practical for aircraft to achieve the net benefit in flight.

Case 1: Inlets for Air Injection

When injection air is provided by jet inlets or other inlets, additional ram drag $\Delta D_{\mathrm{ram}} = V_\infty \Delta\dot{m}_0$ is generated, and the change of the mass flow rate $\Delta\dot{m}_0$ is equal to the rate of air injection $\rho_w V_w S_w$. Thus, the coefficient of the ram drag reduction is given by

$$\Delta C_{D_{\mathrm{ram}}} = 2\left(\frac{\rho_w}{\rho_\infty}\right)\left(\frac{V_w}{V_\infty}\right)\left(\frac{S_w}{S_{\mathrm{wing}}}\right) = \bar{\xi}\bar{C}_{f,0}\left(\frac{\rho_e}{\rho_\infty}\right)\left(\frac{V_e}{V_\infty}\right)\left(\frac{S_w}{S_{\mathrm{wing}}}\right) \tag{8}$$

For $\rho_e = \rho_\infty$ and $V_e = V_\infty$, the change of the total drag coefficient is

$$\Delta C_D = \Delta C_{D_f} + \Delta C_{D_{\mathrm{ram}}} = \bar{C}_{f,0}(S_w/S_{\mathrm{wing}})[f(\bar{\xi}) + \bar{\xi} - 1] \tag{9}$$

Because $f(\bar{\xi}) + \bar{\xi} - 1 \ge 0$, the total drag is actually increased. Certainly, the weight criterion Eq. (3) cannot be satisfied, and this flow control arrangement does not work practically.

Case 2: Rear Suction for Air Injection

An ideal case is considered here. Air suction is used in the rear part of a fuselage to provide injection air for skin friction reduction in the main section of the fuselage. Although suction may reduce the pressure drag, we do not take it into account in this example. The control power includes the powers for injection and suction. The available control power of injection through a porous/perforated wall is given by $P_{\mathrm{FC,inj}} = |\Delta p_w| S_w V_w$, where $|\Delta p_w|$ is the pressure difference across the porous/perforated wall and S_w is the area of the porous/perforated wall. The quantities $|\Delta p_w|$ and V_w are assumed to be uniform over the area S_w. Similarly, the power available for air suction is $P_{\mathrm{FC,suc}} = |\Delta p_{\mathrm{suc}}| S_{\mathrm{suc}} V_{\mathrm{suc}}$, where $|\Delta p_{\mathrm{suc}}|$, S_{suc} and V_{suc} are the pressure difference, area, and velocity for suction, respectively. When the mass flow rate for injection is equal to that for suction (i.e., $\rho_w S_w V_w = \rho_e S_{\mathrm{suc}} V_{\mathrm{suc}}$), the power available for injection and suction together is $P_{\mathrm{FC},a} = P_{\mathrm{FC,inj}} + P_{\mathrm{FC,suc}} = (|\Delta p_w| + |\Delta p_{\mathrm{suc}}|) \times (\rho_w/\rho_e) S_w V_w$. Using a simpler notation, we have $P_{\mathrm{FC},a} = \Delta p S_w V_w$, where $\Delta p = (|\Delta p_w| + |\Delta p_{\mathrm{suc}}|)(\rho_w/\rho_e)$ is the representative pressure difference for injection and suction together. Introducing the power density $\eta_{\mathrm{FC},a} = P_{\mathrm{FC},a}/\Delta W$ available for flow control, we have the normal injection velocity $V_w = \eta_{\mathrm{FC},a}\Delta W/(\Delta p S_w)$ and the parameter

$$\bar{\xi} = \frac{\rho_w}{\rho_e}\frac{2}{\bar{C}_{f,0}}\frac{\eta_{\mathrm{FC},a}\Delta W}{\Delta p S_w V_\infty} \tag{10}$$

For a turbulent boundary layer on a cylindrical fuselage, when the boundary-layer thickness is much smaller than the fuselage diameter, the skin friction formula $\bar{C}_{f,0} = 0.074/Re_{l_W}^{1/5}$ for a flat-plate boundary layer can be approximately used [5], where $Re_{l_W} = V_\infty l_w/\nu$ is the Reynolds number based the longitudinal length $l_w = S_w/\pi d_0$ of the cylindrical fuselage, and d_0 is the maximum diameter of the fuselage. The length l_w is related to the fuselage body length l_{body} by $l_w = R_A l_{\mathrm{body}}$, where $R_A = S_w/S_{\mathrm{body}}$ is the ratio between the injection area S_w and total fuselage body surface area $S_{\mathrm{body}} = \pi d_0 l_{\mathrm{body}}$. Instead of studying a specific aircraft, we consider a canonical aircraft that is a statistical representative of a large group of aircraft, following the scaling laws on the geometrical and aerodynamic parameters based on the mean aircraft weight [6]. These are the scaling laws for propeller/turboprop aircraft: $d_0 = 0.0481W^{1/3}$, $l_{\mathrm{body}} = 0.41W^{1/3}$, $S_{\mathrm{wing}} = 0.0262W^{2/3}$, $\bar{c} = 0.0567W^{1/3}$, and $Re_{\bar{c}} = 6.2 \times 10^4 W^{1/2}$, where $\bar{c}$ is the mean wing chord. In the scaling laws, the lengths are in meters and the weight is in newtons. Using these scaling laws, we have $Re_{l_W} = 4.49 \times 10^5 R_A W^{1/2}$, $S_{\mathrm{body}} = 0.062W^{2/3}$, and $S_w/S_{\mathrm{wing}} = 2.366R_A$. Further, we obtain

$$\Delta C_{D_f} = 0.013 R_A^{4/5} W^{-1/10}[f(\bar{\xi}) - 1] \tag{11}$$

and

$$\bar{\xi} = 378\frac{\rho_w}{\rho_e}\frac{\eta_{\mathrm{FC},a}(\Delta W/W)W^{4/15}}{\Delta p R_A^{4/5}} \tag{12}$$

Similarly, using the scaling laws $V_\infty = 15.52W^{1/6}$ for the cruise velocity and $P_R = 1.67W^{7/6}$ for the cruise power (in which the velocity is in m/s, the power is in watts, and the weight is in newtons), we have the statistical estimates $C_D = 0.0341/\rho_\infty$ and $C_L = 0.3169/\rho_\infty$ for a large group of aircraft. For zero-lift enhancement, substitution of the preceding estimates into Eq. (4) leads to the following weight criterion

$$\begin{aligned}&g(\Delta W/W, W, \eta_{\mathrm{FC},r}, \eta_{\mathrm{FC},a}, \Delta p, R_A, \rho_w/\rho_\infty, \rho_\infty) \\ &= \frac{0.3268\rho_\infty R_A^{4/5}[1 - \exp(-\bar{\xi}/2)]}{(1 + 0.1683\eta_{\mathrm{FC},r}W^{-1/6})W^{1/10}} - \frac{\Delta W}{W} > 0\end{aligned} \tag{13}$$

The function g is interpreted as a benefit margin for flow control by normal injection, which depends on a number of the parameters. The effect of the mean aircraft weight or the size effect on the benefit margin is particularly interesting.

Figure 1 shows the benefit margin g as a function of W for different values of the pressure difference Δp for $R_A = 0.5$, $\Delta W/W = 0.01$, $\eta_{\mathrm{FC},a} = \eta_{\mathrm{FC},r} = 5$ W/N, and $\rho_w = \rho_e = \rho_\infty = 1.21$ kg/m^3. A reduction in the power required for cruise flight can be achieved when the mean aircraft weight exceeds a certain value at which the benefit margin becomes positive. A larger aircraft will benefit more from flow control by normal injection. Similarly, the effects of R_A, $\Delta W/W$, and $\eta_{\mathrm{FC},a}/\eta_{\mathrm{FC},r}$ on the benefit margin are shown in Figs. 2–4, respectively, whereas other parameters remains unchanged. The benefit margin for drag reduction is improved with the increased

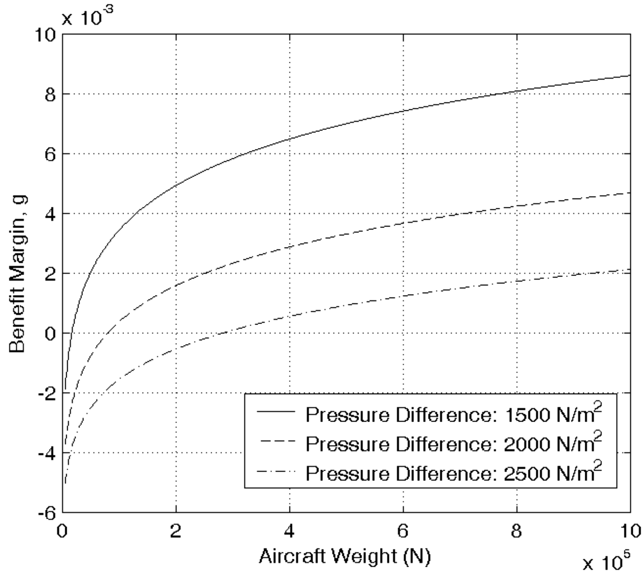

Fig. 1 The benefit margin g at three different values of Δp for flow control by normal injection for $R_A = 0.5$, $\Delta W/W = 0.01$, $\eta_{FC,a} = \eta_{FC,r} = 5$ W/N, and $\rho_w = \rho_e = \rho_\infty = 1.21$ kg/m^3.

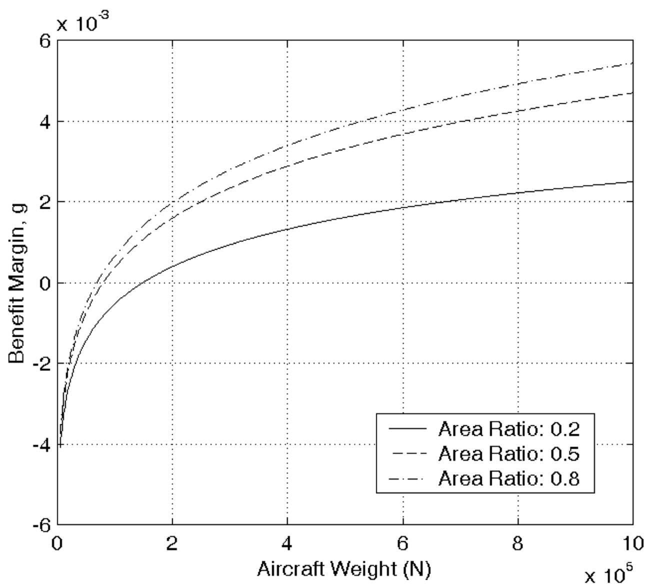

Fig. 2 The benefit margin g at three different values of R_A for flow control by normal injection for $\Delta W/W = 0.01$, $\eta_{FC,a} = \eta_{FC,r} = 5$ W/N, $\Delta p = 2000$ Pa, and $\rho_w = \rho_e = \rho_\infty = 1.21$ kg/m^3.

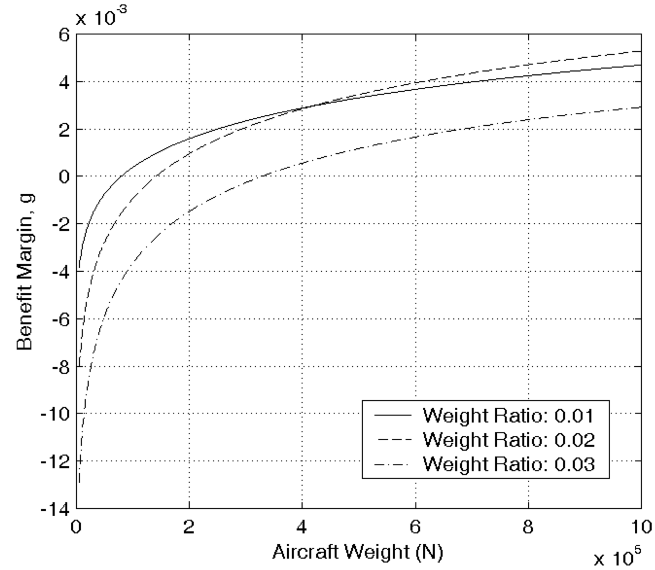

Fig. 3 The benefit margin g at three different values of $\Delta W/W$ for flow control by normal injection for $R_A = 0.5$, $\eta_{FC,a} = \eta_{FC,r} = 5$ W/N, $\Delta p = 2000$ Pa, and $\rho_w = \rho_e = \rho_\infty = 1.21$ kg/m^3.

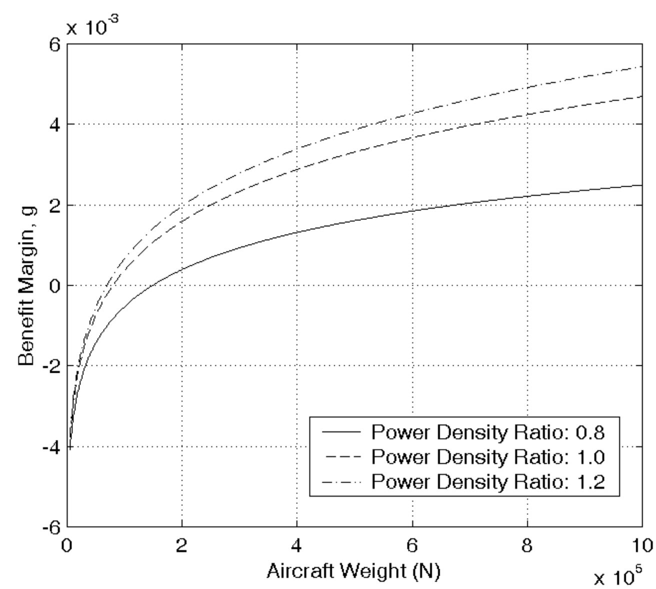

Fig. 4 The benefit margin g at different values of $\eta_{FC,a}/\eta_{FC,r}$ for flow control by normal injection for $\Delta W/W = 0.01$, $R_A = 0.5$, $\eta_{FC,r} =$ 5 W/N, $\Delta p = 2000$ Pa, and $\rho_w = \rho_e = \rho_\infty = 1.21$ kg/m^3.

injection area ratio R_A and power density ratio $\eta_{FC,a}/\eta_{FC,r}$ and decreased control weight ratio $\Delta W/W$.

Conclusions

To achieve a reduction of the power required for level flight, a generic weight criterion is given that can be used in the design and evaluation of a practical flow control system. Two cases are investigated for drag reduction by normal injection. In case 1, in which injection air comes from inlets, an increase in the ram drag overweighs the skin friction drag reduction and the weight criterion cannot be satisfied. In case 2, in which suction near the rear portion of a fuselage provides injection air for skin friction drag reduction, the weight criterion is given in an analytical expression, depending on a number of the parameters such as the mean aircraft weight, relative control system weight, pressure difference, injection area ratio, and control power density ratio. To overcome the weight penalty, the net benefit of drag reduction is more easily achieved for a sufficiently large aircraft. This approach can be similarly applied to other flow control techniques such as synthetic jet and plasma actuator, as long as the functional relationship between the control objectives (e.g., drag reduction and lift enhancement) and relevant parameters is known.

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